Title: Teachable Tessellations

Brief Overview:

This performance based learning unit integrates geometric problem solving with real-life application. Students will demonstrate their knowledge of tessellations and the use of slides, flips, and turns. Students will work cooperatively to design a permanent tessellation activity area for the school blacktop. They will then write a persuasive business letter to the principal explaining why a tessellation blacktop area is a worthy project.

Links to Standards:

- **Mathematics as Problem Solving**
  Students will demonstrate their ability to solve problems in mathematics including problems with open-ended answers, problems which are solved in a cooperative atmosphere, and problems which are solved with the use of technology.

- **Mathematics as Communication**
  Students will demonstrate their ability to communicate mathematically. They will read, write, and discuss mathematics with language and the signs, symbols, and terms of the discipline.

- **Mathematics as Reasoning**
  Students will demonstrate their ability to reason mathematically. They will make conjectures, gather evidence, and build arguments.

- **Mathematical Connections**
  Students will demonstrate their ability to connect mathematics topics within the discipline and with other disciplines.

- **Estimation**
  Students will demonstrate their ability to apply estimation strategies in computation, with the use of technology, in measurement, and in problem solving.

- **Geometry and Spatial Sense**
  Students will demonstrate their ability to apply geometric relationships using one, two, and three dimensional objects. They will demonstrate congruency, similarity, symmetry, and reflection and apply these concepts to the solution of the geometric problems.

- **Measurement**
  Students will demonstrate and apply concepts of measurement using non-standard and standard units and metric and customary units. They will estimate and verify measurements. They will apply measurement to interdisciplinary and real-world problem-solving situations.

- **Patterns and Relationships**
  Students will demonstrate their ability to recognize geometric relationships and will generalize a relationship from data.

Grade/Level:

Grades 3-4

Duration/Length:

This learning unit will take approximately 4 periods (45-60 minutes each)

Prerequisite Knowledge:

Students should have working knowledge of the following skills:

- estimating, rounding money to the nearest dollar, and place value
- recognize and identify polygons
- area
- perimeter
- business letter format
- experience with persuasive writing

Objectives:

Students will:

- work cooperatively in groups**.**
- demonstrate geometric patterns and movement: slides, flips, and turns.
- determine which regular polygons tessellate.
- design a unit cell using regular polygons and create a tessellation from this unit cell.
- be able to use various strategies to solve problems.

Materials/Resources/Printed Materials:

- Pattern blocks (for students and overhead
- White paper (small and large)
- Blank transparencies or leftover lamination film (optional)
- Overhead projector and overhead markers (optional for student use)
- Pencils
- Chart paper and markers
- Pictures, photographs, paintings, posters, tile work, etc. as real-life examples of tessellations (i.e., beehive, armadillo's shell, patios, turtle's shell, brick walls)
- Coloring utensils (crayons, colored pencils, etc.)
- Teacher Resource Sheets: Tessellations.TR1, Tessellations.TR2, Tessellations.TR3, Tessellations.TR4, Tessellations.TR9, Tessellations.TR12, Tessellations.TR15, Tessellations.TR16, and Tessellations.TR17.
- Student Resource Sheets: Tessellations.SR5, Tessellations.SR6, Tessellations.SR7, Tessellations.SR8, Tessellations.SR10, Tessellations.SR11, Tessellations.SR13, and Tessellations.SR14.
- Transparencies of the following teacher and student resources: TR2, TR3, TR4, SR5, SR6, SR7, SR8, SR10, SR14,
- Assortment of materials (contact paper, cardboard, art paper, etc.)
- Access to lamination machine (optional)
- Permanent markers (optional)

Development/Procedures:

Day 1:

- Distribute pattern blocks to each cooperative pair of students.
- Use EPR (Every Pupil Response) to review pattern block shapes.
- Direct cooperative pairs to create any design using the yellow, red, green, and blue pattern blocks making sure there are no gaps/spaces between pieces. Pieces should touch on edges, not only on vertices. (See figure 1, TRY)
- OPTIONS: 1) Have students trace around each block on plain white paper and add color to their design. 2) Have students trace around each block on a blank transparency or leftover lamination film) with overhead projector.
- Instruct students to determine the perimeter and area of each design. Brainstorm with students the unit of measurement they will use (i.e., cm, inches, "green unit") and how to label perimeter vs. area. (See figure 2, TRY--use SR18)
- Conduct a "gallery walk" so students can view other students' designs.
- Facilitate discussion using open-ended questions in reference to students' designs looking for patterns, symmetry, similarities, and discoveries.
- Inform students that they just "tessellated". Display transparency of TR2 and explain the meaning of tessellations.
- Have students TPS (Think/Pair/Share) real-life examples of tessellations and use TR3 to Web/list their responses on chart paper for future reference.

- Have students create and trace a space for a glossary into their math journal. Introduce the term “cell” and develop a class definition.
- Display photographs, posters, etc. of examples of tessellations in and around the world. Again, facilitate discussion using open-ended questions looking for patterns, symmetry, similarities, and discoveries.
- Distribute a large piece of white paper to individual students and allow them to continue to use pattern blocks to create and trace other designs with original patterns and lines of symmetry.
- HOMEWORK OPTIONS: 1) Students determine area and perimeter of each design created and add color to their final product. 2) Students draw one of their designs into their math journal, define a tessellation, and explain why their design is an example of a tessellation. 3) Students gather examples of tessellations (magazine pictures, photographs, etc.) and bring them to class to add to class bulletin board. Students can also explain in class orally or in a sentence why their picture is an example of a tessellation. 4) SR19 5) SR20 6) SR21

Day 2:

- Display transparency of TR4 which shows three different example of tessellations depiction either a flip, slide, or turn. Elicit from students names of the polygons that make each pattern and how each polygon was moved to create this pattern.
- Distribute pattern blocks and SR5. Display a transparency of SR5 on the overhead projector.
- Model how to predict the reflection of the hexagon and then find the actual reflection. Allow students to work in pairs to complete activity.
- Return to transparency or TR4 to show how flips created the pattern in figure A.
- Repeat the same procedure for introducing slides and turns using SR6, SR7, transparencies of SR6 & SR7, and transparency of TR4 figures B & C.
- Distribute SR8 and a pattern template from TR9 to each student. Display TR10 and guide students through activity.
- Display the "Dazzling Designs" finished products in the classroom or bind them into a class book.
- HOMEWORK: SR11

Day 3:

- Review pattern block shapes and characteristics or each (sides, angles, vertices)
- Review flips, slides, and turns across a line of symmetry.
- Revisit Web created on previous day of real-life examples of tessellations. Add new discoveries to Web.
- Discuss students' designs from Days 1 & 2 and the various patterns created.
- Introduce and define the term "cell". Look for examples in students original designs and find cell patterns.
- Use TR12 to elicit predictions of repeating patterns made by simple cells.

- Instruct students to create a cell using at least two shapes. Students should create a "sample draft" of an original design on white paper by tracing their cell pattern. Students should add color to their sample draft.
- Direct students to have their sample draft previewed by the teacher prior to creating their final "assessment" product on the materials of their choice.
- ASSESSMENT OPTIONS: Provide students with an assortment of materials such as contact paper, cardboard, art paper (to be laminated at completion of project), etc. Invite students to transfer their sample draft pattern onto one of the materials to create a desk covering, book cover, work mat, placemat, or some other idea. CRAFT IDEA: Use final products as holiday gifts (Mother's Day, Father's Day, Hanukkah, Christmas, etc.)
- HOMEWORK: SR13

Day 4:

- Present and discuss writing prompt (SR14) to cooperative groups. Challenge students to design a permanent tessellation activity area for the blacktop. Suggest that students can create tessellations on the blacktop using chalk and enlarged pattern blocks during recess (See TR15 for a sample design of what students might choose to create). Inform students they will also write a persuasive letter to the principal to gain his/her support and approval for this culminating project.
- Monitor groups as they design their blacktop activity area, stopping to offer assistance and to ask open-ended questions for focusing of ideas.

Performance Assessment:

Assessment of student progress is ongoing throughout this unit. The activity pages include "challenges" for students to demonstrate their understanding of new concepts. Teacher Resource 16 is a scoring rubric for assessing the persuasive business letter. Students will be expected to work cooperatively throughout this unit. The use of observation, checklists, and anecdotal records may be used to assess students as they complete the activities/tasks outlined in the learning unit. Students may contribute to the generating of criteria to evaluate their final tessellation project. Written responses to activities may also be used to assess students' understanding.

Extension/Follow Up:

- Actively pursue implementation of playground design by presenting proposal letters to principal
- Further research cost amounts for materials needed to implement design and calculate a more accurate total cost.
- Tessellate with various materials to make a mural, bulletin board, or quilt.
- Tessellate a decorative border for the classroom on white contact paper.
- Make large "cells" out of wood or some other durable material to be used at recess.

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FIGURE 1

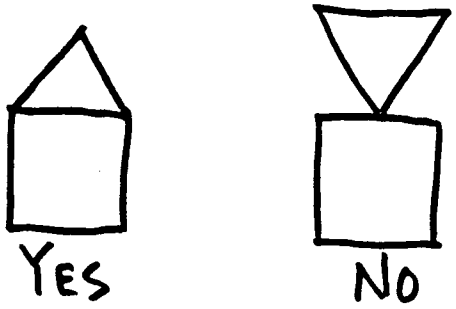
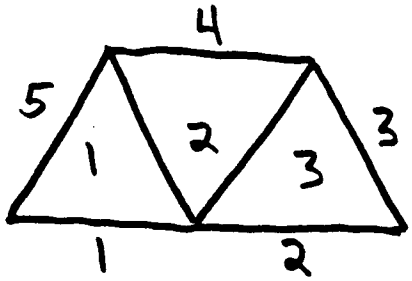


FIGURE 2

green = unit of measure

Perimeter = green units
Area = triangular units



Perimeter = 5 green side length units
Area = 3 triangular units

Created by Bettina Oliver, Howard County Public Schools, Ellicott City, Maryland.

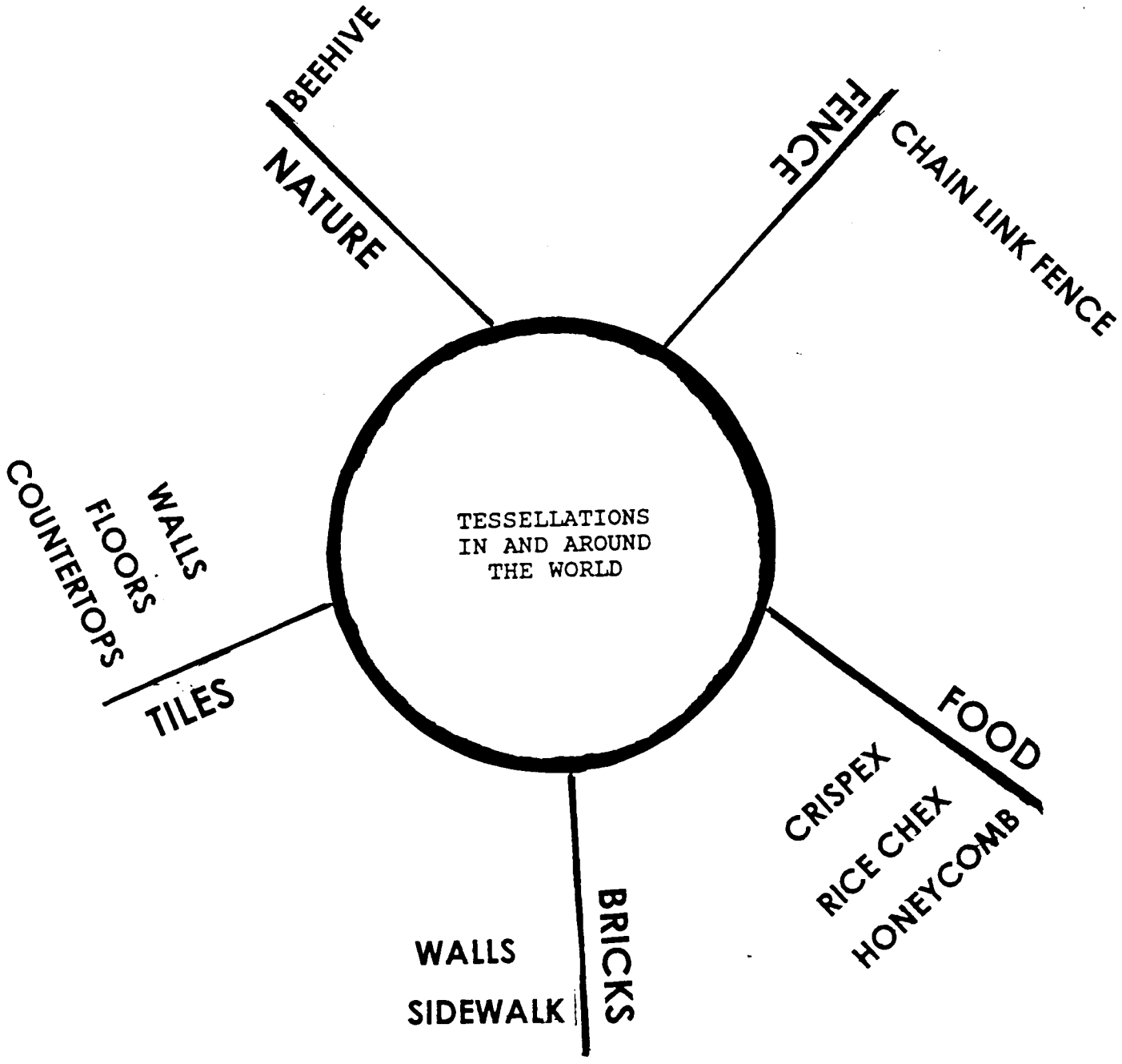
Tessellations

Tessellations comes from the Latin word **tessella** - the small, square tile used in ancient Roman mosaics.

A tessellation is a tiling, made up of a repeated use of a shape. These repeated shapes completely fill a plane without any gaps or overlaps.

The Dutch artist, ESCHER, is famous for hundreds of tessellations during his lifetime (1890 - 1972).

Shared by Kay Sammons, Instructional Facilitator, Howard County Public Schools, Ellicott City, Maryland



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TESSELLATION TRIO

FIGURE A

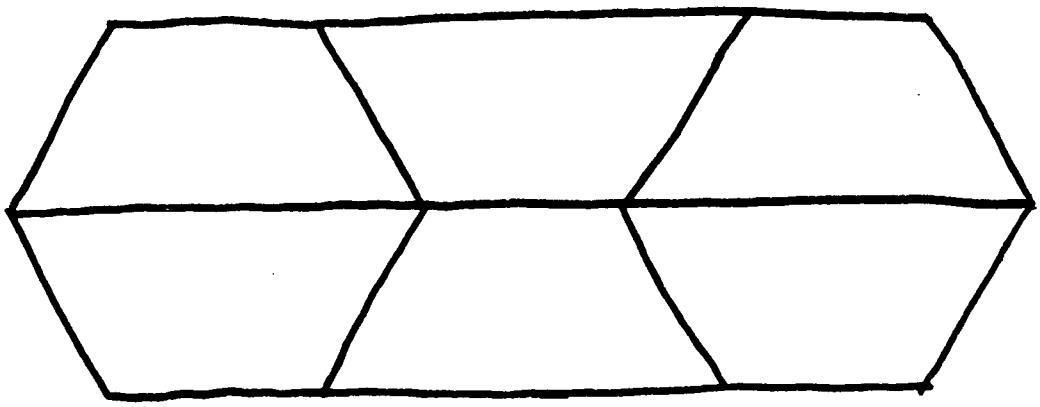


FIGURE B

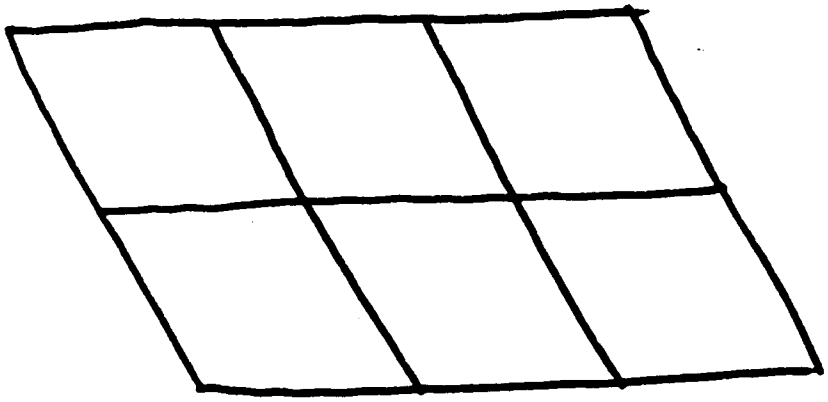
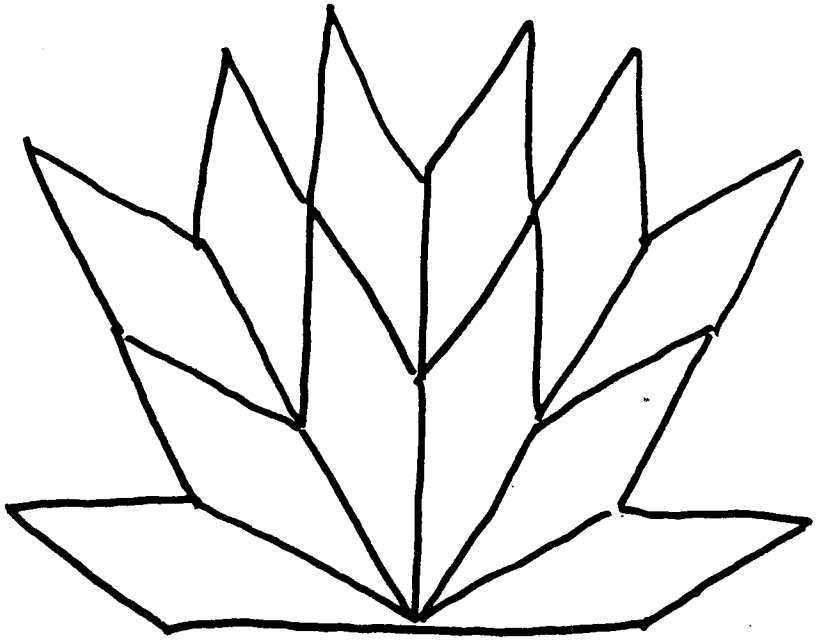


FIGURE C



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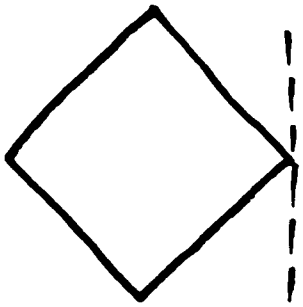
NAME_____ DATE_____

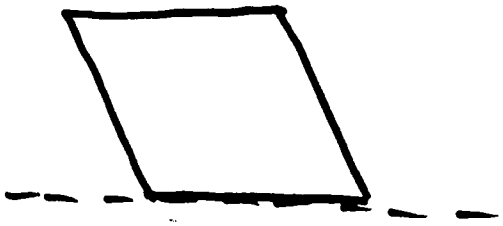
FLIP FLOP!

When a shape is flipped across a line of symmetry it makes a "reflection" image. Predict what each shape's reflection will look like when flipped over the line and sketch your prediction in pencil. Use your pattern blocks to find and trace the actual reflection.

1)

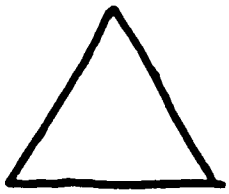


2)



FLIP FLOP CHALLENGE

How many different reflections can you make by flipping this polygon?



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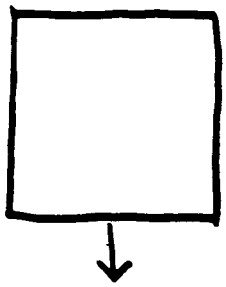
NAME_____ DATE_____

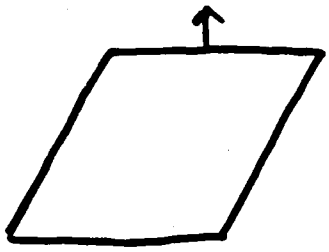
SLIP & SLIDE

When a shape slides across a flat surface it looks exactly the same as it did in its starting position. Predict what each shape will look like after it slides across a surface and sketch your prediction in pencil. Use your pattern blocks to find what each shape actually looks like after it slides across the surface.

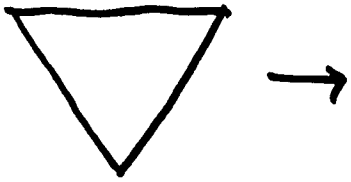
1)

2)



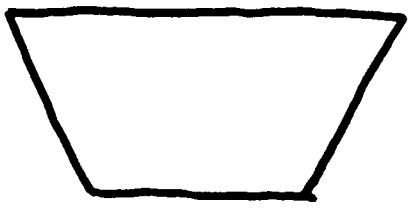


3)



SLIP & SLIDE CHALLENGE

Can you draw more than three ways to slide this shape?



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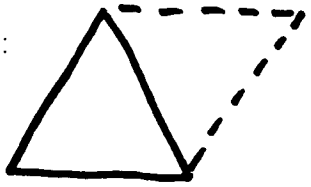
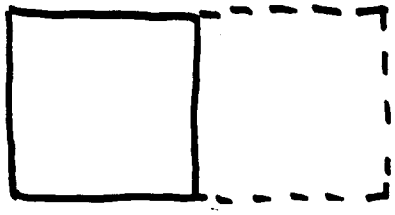
NAME_____ DATE_____

AS THE SHAPE TURNS

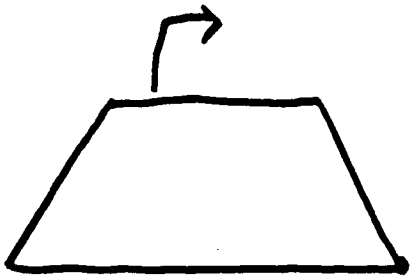
When shapes are turned by rotating on a stationary point the shape looks exactly the same and is in a new location. Predict what each shape will look like after it turns to a new position and sketch your prediction in pencil. Use your pattern blocks to find what each shape actually looks like after it turns to a new position.

EXAMPLES:





1)



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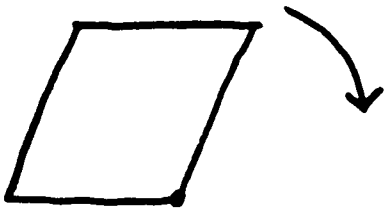
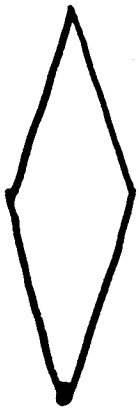


AS THE SHAPE TURNS CHALLENGE

How many turns can be made by rotating this shape without overlapping?



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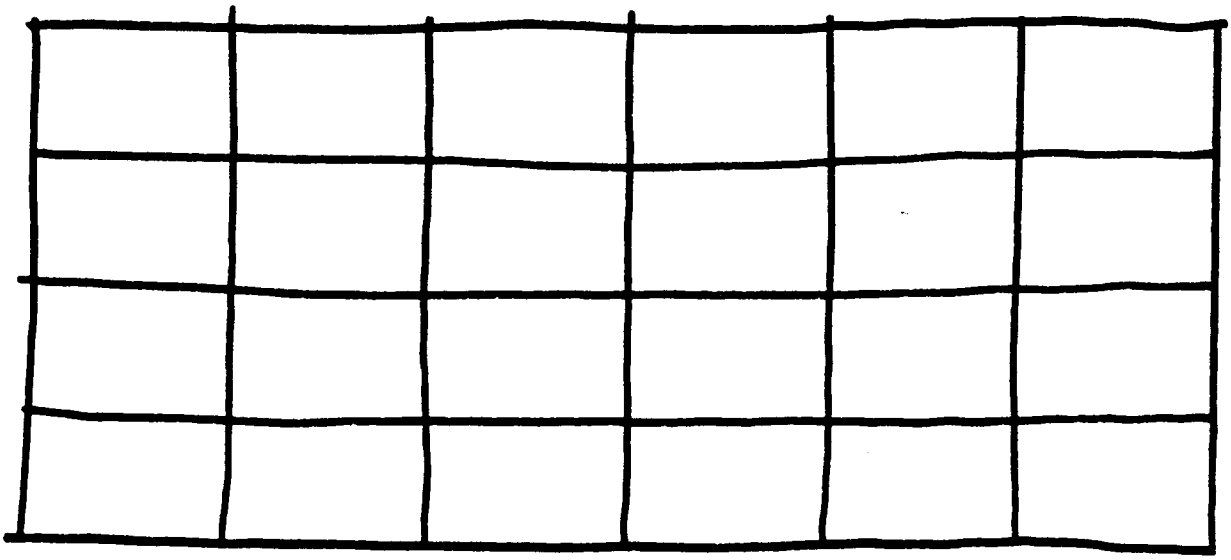
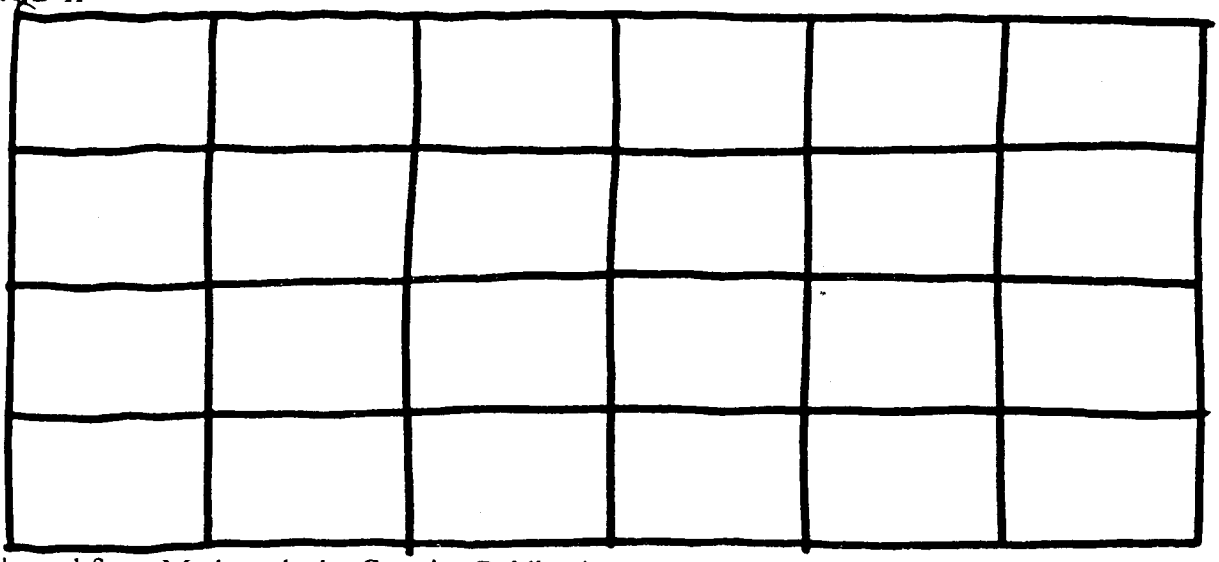
NAME_____ DATE_____

DAZZLING DESIGNS
(Pattern sheet)

GRID I



GRID II

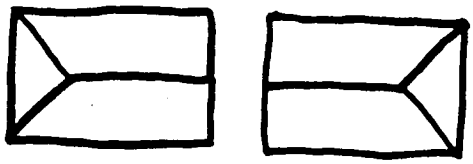


Adapted from Mathworks by Creative Publications.

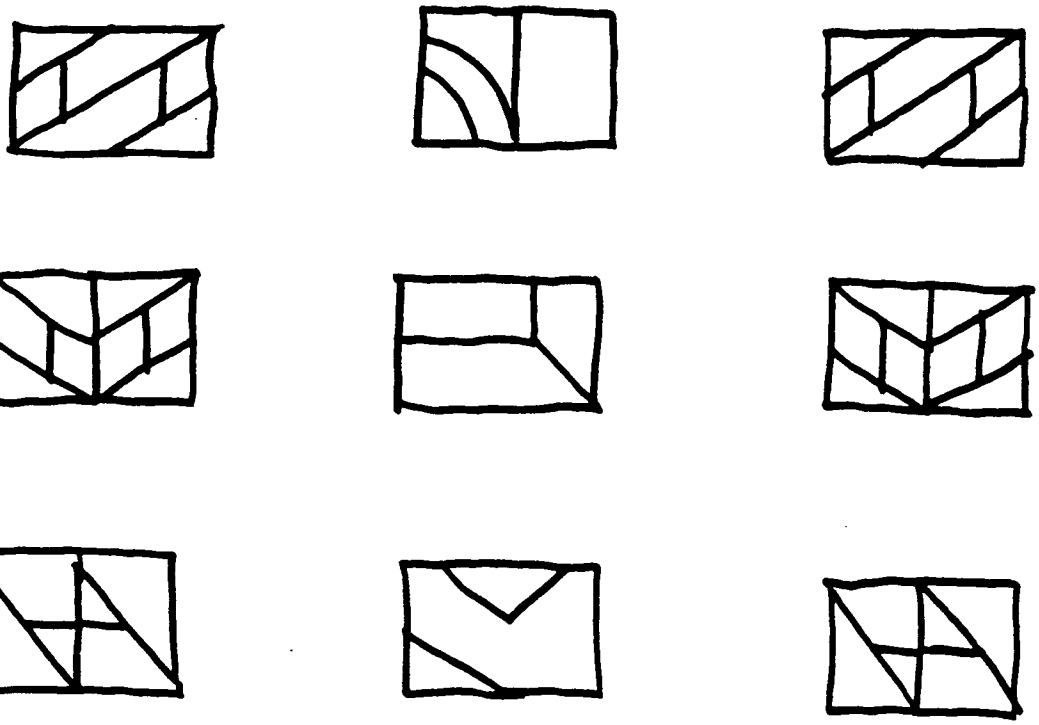
DAZZLING DESIGNS
(Pattern Templates)

1) Duplicate the following pattern templates and cut them out.
2) Draw the reflected pattern on the back of each template as shown below. Pattern lines should be back to back.

EXAMPLE:



FRONT BACK



Adapted from Mathworks by Creative Publications.

DAZZLING DESIGNS
(Directions)

1) Copy the pattern template design in the top left block of grid I.

2) Place the template on top of the copy in the first block and "flip" template over into the box on its right.

3) Copy this flipped design in the second block.

4) Continue flipping and copying in this way until all blocks of row one are completed.

5) Begin the second row by placing the pattern template in the top left block again. Flip template down to create the design of the first block of the second row.

6) Flip and copy this design in the same manner as row one.

7) Repeat steps 5 & 6 to complete rows three and four.

8) Copy the pattern template design in the top left block of grid II.

9) Copy and "slide" the pattern card across the rows or down the columns to produce the designs.

10) Add color to both patterns. Different color arrangements of the same pattern create completely different effects.

Adapted from Mathworks by Creative Publications.

NAME_____ DATE_____

OPERATION LOCATE

We have been exploring how polygons tessellate. To tessellate is to completely cover a flat surface with a repeated pattern without leaving any spaces. We discovered that many unique and interesting patterns could be created by flipping, sliding, and turning shapes.

There are many tessellation patterns located in places we see every day such as floors, wallpaper, fabric, art, and buildings. Locate a tessellation pattern and complete the following activities:

1) Draw the polygon that was used to create the tessellation pattern.

2) Describe where you located the tessellation pattern.

3) Write about how flips, slides, and/or turns were used in the tessellation pattern.

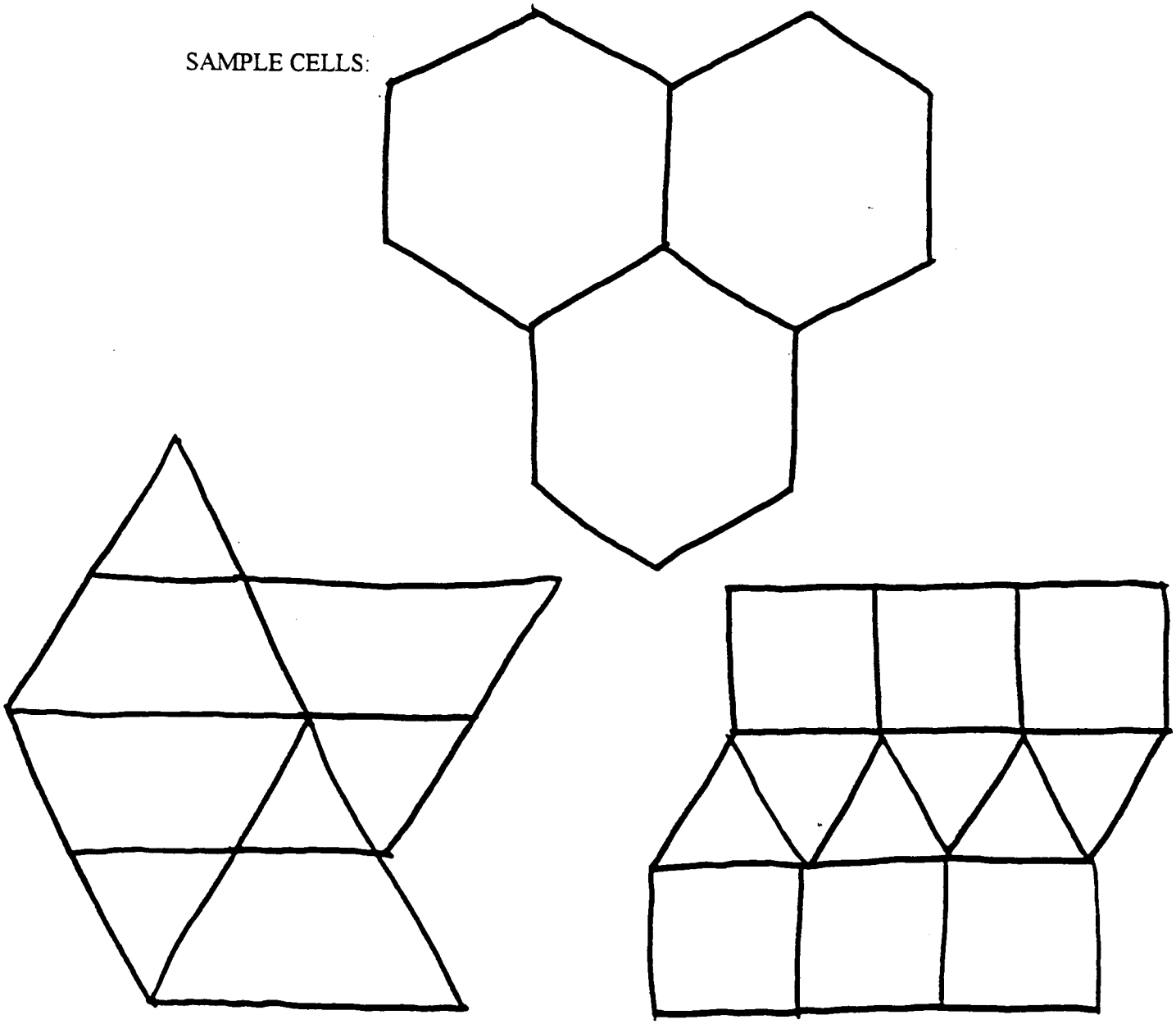
4) Describe the purpose the tessellation served.

Adapted from Nature's Mathematical Marvels by James Burnett and Calvin Irons, Mimosa Publications, 1996

CELL

To create a "cell" use one or more geometric shapes (ie. pattern blocks) to form a repeating pattern with no gaps or spaces. You may incorporate flips, slides, or turns.

SAMPLE CELLS:



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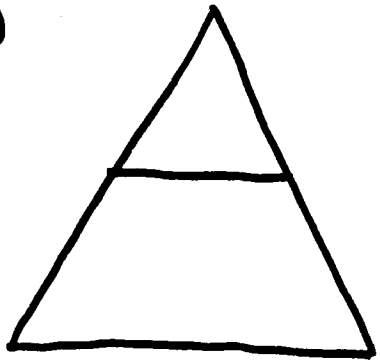
NAME_____ DATE_____

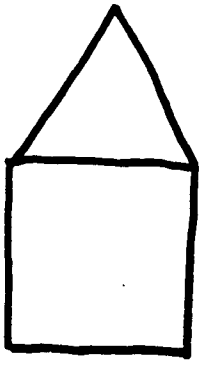
REPEAT A CELL

Look at the patterns below. Guess how the pattern will look if repeated. Trace each cell at least twice to show the tessellation it will create. Label flip, slide, and/or turn to show which movement or movements were used.

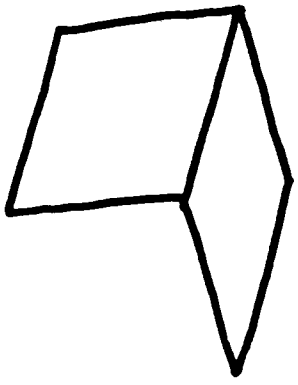
1



2



3



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Tessellations.SR14

PROMPT

Your class has been asked to design an activity area for the school's playground. The purpose of the playground area is to have a specific place to draw tessellations with sidewalk chalk and enlarged pattern block tessellation cells.

You will be divided into cooperative groups to create a playground plan. Using what you know about tessellations, think about how you might want to section off the playground area. You may wish to include permanent sample cells in your area. Remember to plan the necessary materials for your enlarged traceable pattern blocks. Estimate the cost of all your supplies, including paint and chalk.

Now, each of you will write a persuasive letter to your principal, asking permission to paint a permanent tessellation area on the playground. Include a copy of your plan and tell why you think a tessellation area is a worthy math project. Describe at least three math concepts that will be used while "playing" in the tessellation activity area. Since your letter will be read by the principal, be sure to edit for CUPS (Clear ideas, Usage of grammar, Punctuation and capitalization, and Spelling).

Remember to use “FAT P” (Form, Audience, Topic, Purpose)

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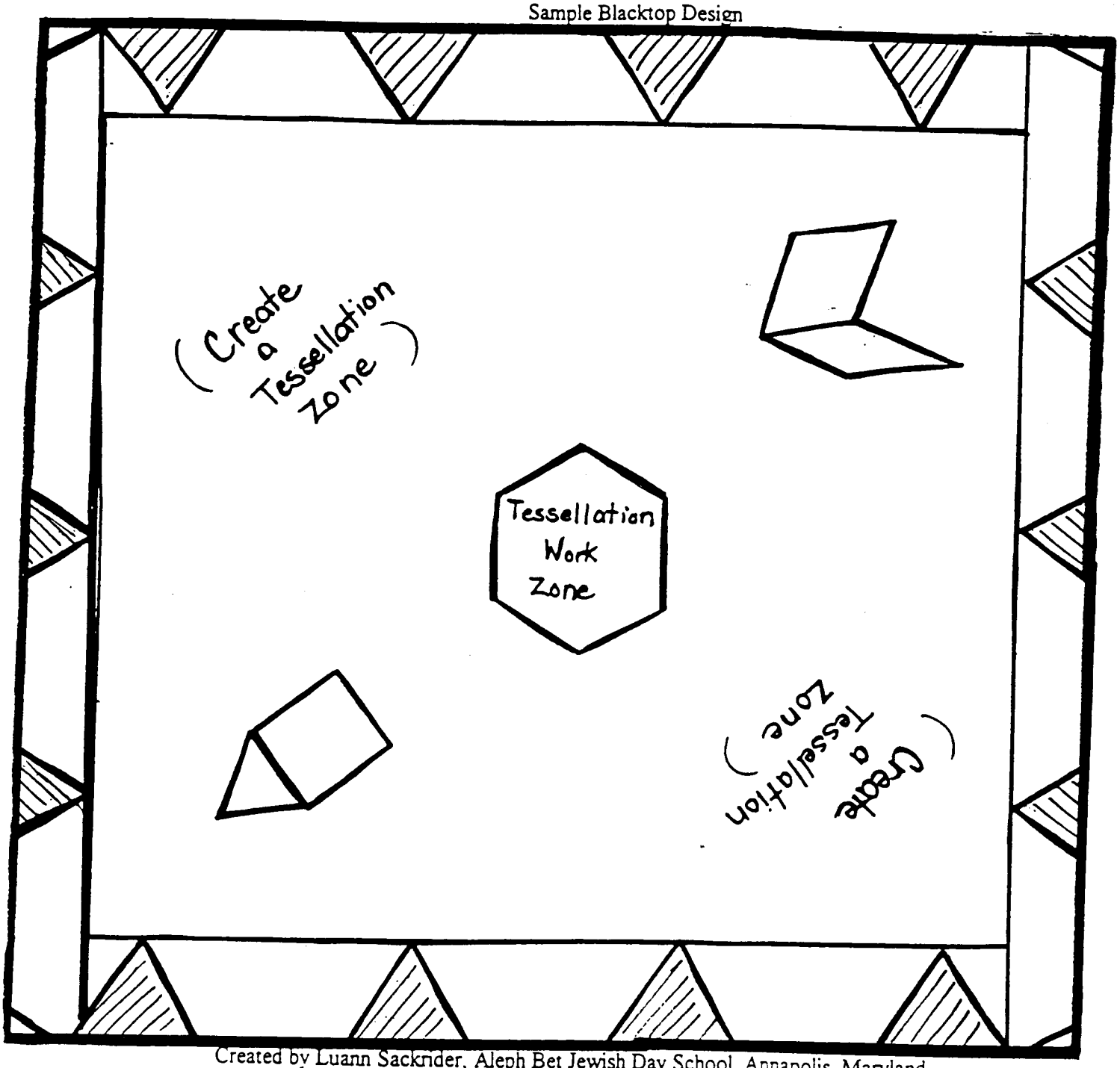
Sample Blacktop Design



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RUBRIC

Think about these things while scoring the Tessellation Project and Formal Persuasive Letter:

* Did the student work in a cooperative group to create a playground plan?
* Did the student use repeating cells in the plan to form a tessellation?
* Did the student demonstrate reasonable estimation skills?
* Did the student ask permission to make a permanent tessellation area?
* Did the student describe at least three math concepts that would be used in the activity area?

Scoring Rubric:

4 Point Score:

The student worked cooperatively and demonstrated exceptional understanding of tessellations. The student included three math benefits which others would gain from using the new activity area in his/her persuasive letter.

3 Point Score:

The student worked cooperatively and demonstrated good understanding of tessellations. The student included three math benefits others would gain from the new activity area in their persuasive letter.

2 Point Score:

The student worked cooperatively and demonstrated a fair understanding of tessellations. The student included two math benefits others would gain from the new activity area in their persuasive letter.

1 Point Score:

The student worked cooperatively and demonstrated some understanding of tessellations. The persuasive letter did not include two-three strong arguments.

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GLOSSARY

area- the surface space enclosed within a boundary.

cell- to use one or more geometric shapes (i.e., pattern blocks) to form a repeating pattern with no gaps or spaces.

estimate- predict an approximate quantity or size of a given sample.

flip- to turn a geometric shape over a line of symmetry making a reflecting image.

hexagon- a six-sided polygon with six angles.

perimeter- the distance around the outside of a two-dimensional shape.

polygon- a two-dimensional shape with straight sides.

predict- making an educated "guess" based on prior knowledge.

rectangle- a four sided polygon with two pairs of opposite sides parallel and four right angles.

regular polygon- a polygon that has sides of equal length and all angles equal.

rhombus- a four sided polygon with two pairs of opposite sides parallel and of equal length.

slide- to move a geometric shape in a straight line across a flat surface to make a repeating pattern.

symmetry- a shape is considered symmetrical when one side is a mirror image of the other side.

tessellation- when two-dimensional shapes fit together to make a repeating pattern without any gaps or overlaps.

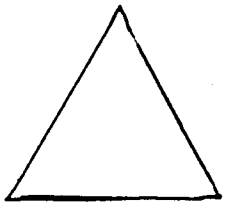
trapezoid- a four sided polygon with one pair of opposite sides parallel.

turn- to move a geometric shape by rotating on a stationary point.

Created by Luann Sackrider, Aleph Bet Jewish Day School, Annapolis, Maryland

NAME: _____

PERIMETER AND AREA

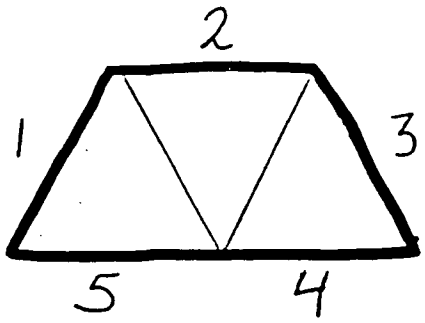


= unit of measure

Perimeter = green units

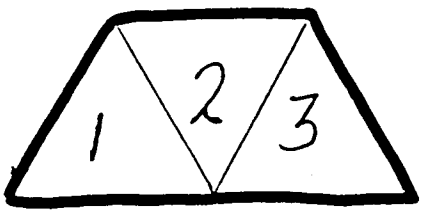
Area = triangular units

Perimeter is the distance around the outside of a two-dimensional shape.



Perimeter = 5 green units

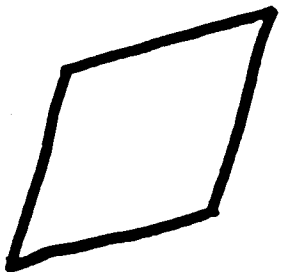
Area is the surface space enclosed within a boundary.



Area = 3 triangular units

For each of the shapes below determine the perimeter and area.

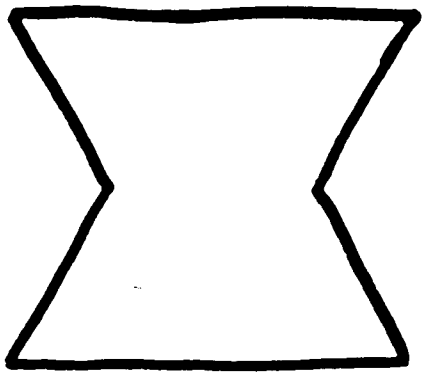
1)



Perimeter = _____

Area = _____

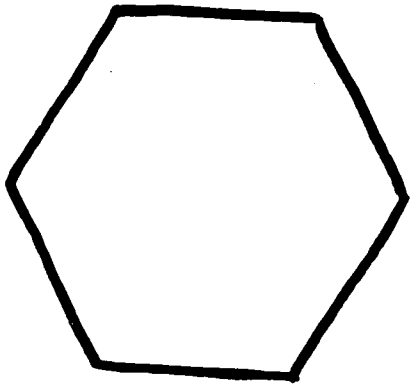
2)



Perimeter = _____

Area = _____

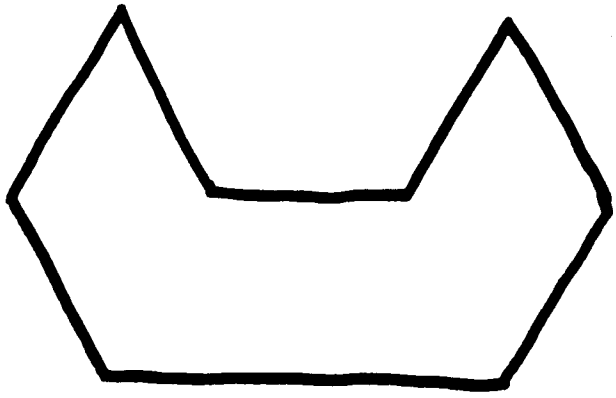
3)



Perimeter = _____

Area = _____

4)



Perimeter = _____

Area = _____

NAME: _____

TESSELLATION HOMEWORK

1) Design your own pattern in the space below.

2) For the above pattern, determine the perimeter and area.

Perimeter = _____ Area = _____

3) Define a tessellation.

_____

_____

_____

4) Why is your design a tessellation?

_____

_____

_____

_____

SLIDES, TURNS, and FLIPS

1) SLIDE

2) TURN

3) REFLECTION (FLIP)